MANDATORY EXERCISE

Linear Elasticity and solid pressure
We will consider the topic 'locking'. Consider the following equation on the domain $\Omega = (0,1)^2$:

$$(1) \qquad -\mu\Delta u - \lambda\nabla\nabla\cdot u = f \text{ in } \Omega,$$

$$(2) \qquad u = u_{analytical} \text{ on } \partial\Omega$$

The equation is known to suffer from locking, i.e. the numerical solution "locks" as λ tends to infinity, meaning that the numerical deformation is much smaller than the actual deformation. (You may replace Δu with $\nabla\cdot(\epsilon u)$.)

Consider for example an analytica solution of the form: $u_{analytical} = (\frac{\partial\phi}{\partial y}, -\frac{\partial\phi}{\partial x})$, where ϕ is given and check if you are able to construct a numerical example that demontrate locking. Clearly, $-\mu\Delta u = f$ as $\nabla\cdot u$ is zero.

Check to what extent the formulation with the solid pressure reduce locking.

Splitting schemes for the Navier-Stokes equations (Newtonian, incompressible)
In the notes/lectures it is stated that common splitting schemes like IPCS and variants can never be of higher order than 1. Explain why.

We have considered two approaches to discretize the Navier-Stokes equations: 1) discretize in time then space or 2) discretize in space before time. The approaches do not always lead to the same boundary conditions. Explain why.

Numerical error of positive PDEs
The Poisson problem, the convection diffusion problem (assuming divergence free velocity) and linear elasticity problem are all example of positive or elliptic PDEs. Explain why.

Show Galerkin orthogonality almost directly leads to estimates of the numerical error (Cea's lemma) for positive PDEs.