MANDATORY EXERCISE

Exercise 1

Consider the following equation on the domain $\Omega = (0,1)^2$:

$$-\mu\Delta u + u_x = 0 \text{ in } \Omega, \tag{1}$$

$$u = 0 \text{ for } x = 0, \tag{2}$$

$$u = 1 \text{ for } x = 1, \tag{3}$$

$$\frac{\partial u}{\partial n} = 0 \text{ for } y = 0 \text{ and } y = 1 \tag{4}$$

a) Derive an expression for the analytical solution.
b) Compute the numerical error for $\mu = 1, 0.3, 0.1$ at $h = 1/8, 1/16, 1/32, 1/64$.
c) Compare against the expected error estimate, that is; assume:

$$\|u - u_h\|_1 \leq C_\alpha h^\alpha$$

and

$$\|u - u_h\|_0 \leq C_\beta h^\beta.$$

That is, the error estimates in the H^1 and $L^2 = H^0$ norms. Estimate C_α, C_β, α and β and check whether the expected error estimate is valid.
d) Implement the Streamwise Upwinding Petrov-Galerkin (SUPG) method and compare against the results in b) and c).

Exercise 2

Consider the famous benchmark of " Schäfer, Michael, et al. "Benchmark computations of laminar flow around a cylinder." Flow simulation with high-performance computers II. Vieweg+ Teubner Verlag, 1996. 547-566".
Boundary conditions. Set no-slip (velocity equal to zero) on walls and cylinder. Set Dirichlet velocity on the inlet, describing a parabolic profile. Let there be homogenous Neuman conditions for the velocity at the outflow. For the pressure, let there be homogenous Neumann everywhere, but for the outflow where a (e.g. homogenous) Dirichlet condition is set.
a) Implement a solver for the benchmark problem in FEniCS based on both a fully explicit time discretization and a semi-implicit discretization. Use piecewise linear for the pressure and both piecewise linear and quadratic elements for the velocity (command line option)

b) Assess the stability requirement of both schemes, i.e., what is C, β in $\Delta t \leq Ch^\beta$ that make sure that the scheme is stable.

c) Compute pressure difference and drag. Assess to what extent the numerical value approaches the true value.

Deadline: March 15. Include code. Typesetting in LaTeX is prefered.
