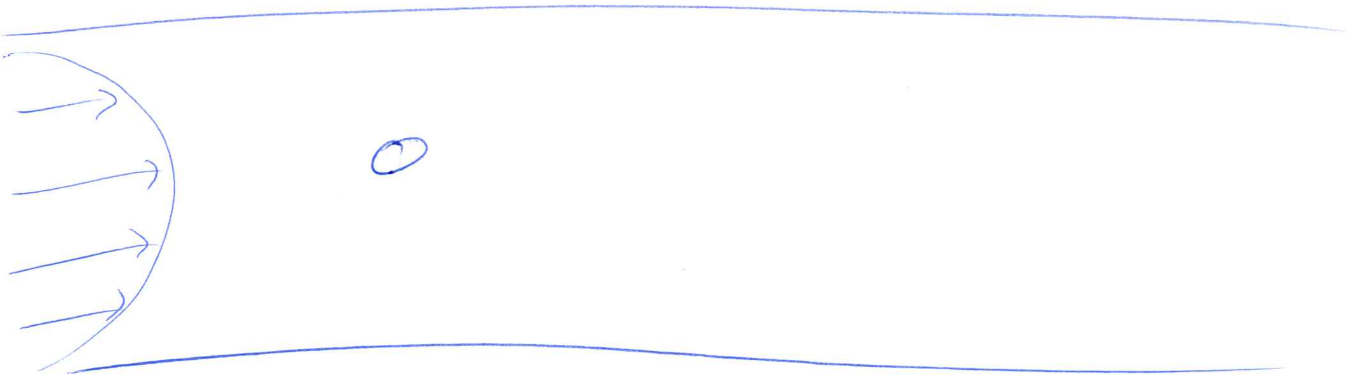
Lecture 11



Earlier we looked at the problem of flow around a cylinder



Some formulated the problem as

$$\frac{\partial v}{\partial t} + (v\cdot\nabla)v = -\nabla p + \boxed{\nu \Delta v} + f$$

$$\nabla\cdot v = 0$$

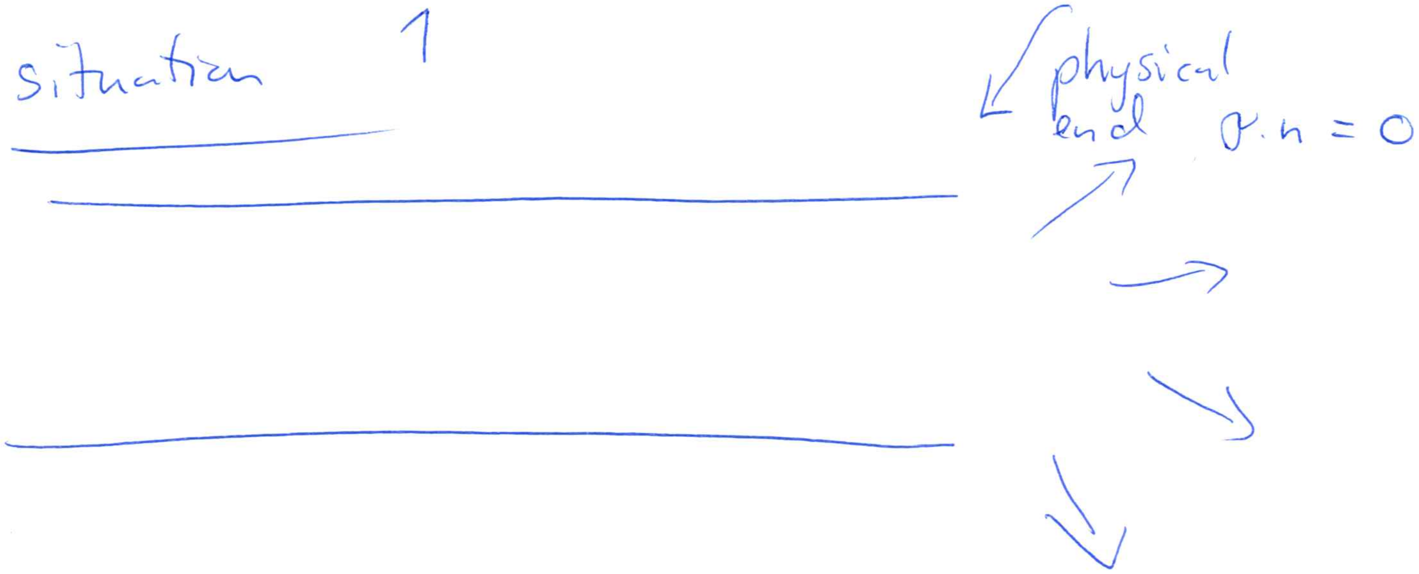
while some did

$$\frac{\partial v}{\partial t} + (v\cdot\nabla)v = -\nabla p + \boxed{\nabla\cdot(\nu\varepsilon(v))} + f$$

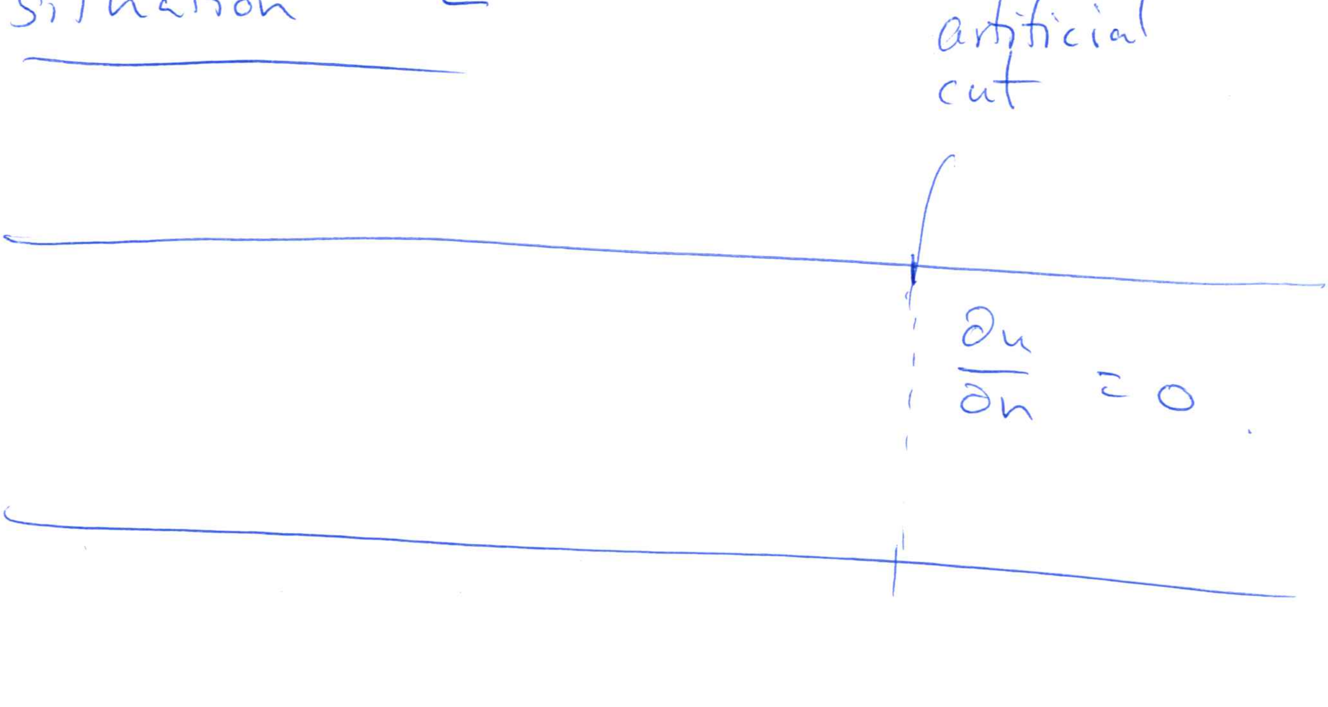
Which is right ?

First, in physics the stress always concerns the symmetric gradient $\varepsilon$
→ the reason is a proper handling of (among other things) rigid motions)

However, our case is special in the sense that the cut on inlet and outlet is artificial situation 1




situation 2



In other words, if I express the equation in terms of stress, but have an ~~ø~~ artificially cut geometry, I may introduce trouble.

**Remedy:**

$\cancel{\int \varepsilon(u):\varepsilon(v)}$

$$-\int_\Omega \nabla\cdot\varepsilon(u)\, v = \int_\Omega \varepsilon(u):\varepsilon(v)$$

$$-\int_{\partial\Omega} \varepsilon(u)\cdot n \cdot v \, ds =$$

$$\int_\Omega \varepsilon(u) : \varepsilon(v) \, dx \;-$$

$$\int_{\partial\Omega} \frac{\partial u}{\partial n} \cdot v \, ds \;-\; \cancel{\int \left(\frac{\partial u}{\partial n}\right)^T \cdot v \, ds}$$

$$\int (\nabla u)^T \cdot n \cdot v \, ds$$

set only this to zero. Keep the other

This is particularly important in multiphysics applications

Question:

is $\int_\Omega \varepsilon(u) : \varepsilon(v) \, dx = \int \varepsilon(u) : \nabla v \, dx$

or in general, if $A$ is symmetric while $B$ is not. Is

$$A : B = A : B_s$$

where $$B_s = \frac{1}{2}\left(B + B^T\right)$$

Answer, yes !!!

Hint check that the inner product between a symetric and anti-symetric matix is zero.